Front Contribution instead of Back Propagation

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Abstract

Deep Learning's outstanding track record across several domains has stemmed from the use of error backpropagation (BP). Several studies, however, have shown that it is impossible to execute BP in a real brain. Also, BP still serves as an important and unsolved bottleneck for memory usage and speed. We propose a simple, novel algorithm, the Front-Contribution algorithm, as a compact alternative to BP. The contributions of all weights with respect to the final layer weights are calculated before training commences and all the contributions are appended to weights of the final layer, i.e., the effective final layer weights are a non-linear function of themselves. Our algorithm then essentially collapses the network, precluding the necessity for weight updation of all weights not in the final layer. This reduction in parameters results in lower memory usage and higher training speed. We show that our algorithm produces the exact same output as BP, in contrast to several recently proposed algorithms approximating BP. Our preliminary experiments demonstrate the efficacy of the proposed algorithm. Our work provides a foundation to effectively utilize these presently under-explored "front contributions", and serves to inspire the next generation of training algorithms.

1 Introduction and Related Work

Backpropagation of error (BP) [28] has been the best algorithm to train neural networks, and has driven deep learning to perform outstandingly across several domains [18]. However, it is not consistent with our findings about the brain [5, 13]. In fact, it is not possible to execute BP in a real brain [2]. BP also suffers from several other problems, such as a vanishing/exploding gradient. Inspite of using careful initialization and architecture modifications [27, 9]– for example, using RELU instead of sigmoid activations– quite effectively as workarounds, BP still is a key bottleneck for memory usage and speed. This may indicate that *BP is a suboptimal algorithm and will be replaced*.

Several different algorithms have been proposed to improve BP. However, those algorithms have tried to approximate BP and have not been able to work beyond toy datasets [2]; thus they cannot be applied in a real world setting. We survey three categories of BP literature– (i) better hardware implementation of BP [15, 16, 31, 11, 32, 25], (ii) workarounds to approximate BP [33, 7, 10], and (iii) biologically inspired algorithms. Biologically inspired algorithms can further be segregated into four types: (i) Inspired from biological observations [29, 7, 26, 17], these works try to approximate BP with the intention resolve its biological implausibility, (ii) Propagation of an alternative to error [19, 21], (iii) Leveraging local errors, the power of single layer networks, and layer wise pre-training to approximate BP [24, 23, 3], (iv) Resolving the locking problem using decoupling [14, 6, 12, 1, 20] and its variants [27, 8, 22, 4]. We were deeply motivated by (ii), (iii), and (iv) while coming up with the idea of 'front contributions' – specifically, propagating something other than error, the idea of a single layer network, and decoupling, collectively inspire 'front contributions'. The key distinction of our front contribution is that, it produces exact same output as BP unlike other approaches that have tried to approximate BP.

2 Method

We know that a set of linear layers can be collapsed to a single layer network; however, non-linear activation functions have hitherto restricted the collapse of networks that they are applied in. Here, we justify that every fully connected multi-layer network can be collapsed to a single layer network, using the *Front Contribution Algorithm*, eliminating the requirement of backpropagation ¹.

Formalization:

Let a neural network (NN) have n layers, such that each layer of its weights are represented as $W_i : i\epsilon[1, n]$, where W_1 is applied to the input and W_n is on the branch connected to the output node. Let the input of NN be X, intermediate layer outputs be $v_j : j\epsilon[1, n-1]$, and final output be Y. We can represent Y as:

$$Y = f(X, W_1, W_2, \dots, W_n)$$
(1)

Here, each of the weights are randomly initialized, and then updated using conventional back propagation. Using the chain rule, we have:

$$\Delta W_1 = f(W_2, W_3 \dots W_n) \tag{2}$$

$$\Delta W_2 = f(W_3, W_4 \dots W_n) \tag{3}$$

From equation 6, we see that the input X is multiplied with the weights, and then transformed by activations at various layers to produce the output Y. In other words, we can say that weight layers $W_1...W_n$ indirectly contribute towards deciding what Y will be for a given X, in a hierarchical sequence starting from W_1 till W_n . The definition of 'contribution' varies depending on the application–for example, in the case of language models like BERT, contribution refers to attention[30].

Now, from equations 7,8 we see that in the training process, for the i^{th} layer, Δ W is a function of all the W_i layers, from the $(i + 1)^{th}$ layer up till the n^{th} layer. From equation 7, we see that ΔW_1 is a function of $W_2...W_n$, but does not further depend on W_1 , i.e., the value of W_1 at any iteration of training depends on the initial value of W_1 - a random static value- and weights $W_2...W_n$. So, we can say that W_1 is not part of the system basis- the set of vectors that can be used to represent any system state, such as the use of x, y, z bases to represent any state of a 3D system- as $W_2...W_n$ can represent the output value at any iteration. So, vector W_1 is not actually a necessary variable.

For example, by defining a 3D system in terms of x, y, z, x + y + z, we utilize an unnecessary variable, x + y + z. As W_1 is not an independent weight layer, we are therefore wasting GPU space by using it. However, if the weight layer W_1 is ignored, will the system still function as desired? The value of v_1 depends on weights in W_1 , so if W_1 weights are not updated, v_2 will retain its old, incorrect value, consequently affecting Y even if all other weights are updated correctly, as the network is connected as a hierarchy.

How do we compensate the non-updation of weights in W_1 ? Let a compensation weight p be added to weights in W_2 such that Y remains the same, i.e., p compensates for the change in v_2 that normally happens with the updation of W_1 . Here, p must be a non-linear function of weights in W_2 , as it compensates for an update that depends on $W_2...W_n$. p can be derived by equating the v_2 value found post updation of W_1 in conventional backpropgation, and the v_2 value calculated without updating W_1 and replacing weights in W_2 as $W_2 + p$.

3 Finding an Expression for *p*:

Given a 2 layer neural network as shown in Figure 2, where inputs are x_1, x_2 , and a single output Y is produced, if $w_1, w_2, w_3, w_4 \in W_1$ are not updated, then non-linear 'compensation weights' of p, q must be added to $w_5, w_6 \in W_2$ respectively in order to preserve Y output updation. Here, we consider outputs at intermediate layer nodes after RELU activation to be v_1, v_2 (for example, $s_1 = w_1x_1 + w_2x_2$; $v_1 = RELU(s_1)$).

¹See Supplementary Material: Analogy for illustrative explanation of backpropagation and front contribution



(b) Forward Contribution



Figure 1: Neural network structure for (a) Backpropagation, (b) Forward Contribution. Here, layer W_1 contains weights w_1, w_2, w_3, w_4 and W_2 contains weights w_5, w_6 .

To derive values² of p, q, consider that for the initial updation, we can say that:

$$Y_{backpropgation} = Y_{contribution factor} \implies v_1.w_5 + v_2.w_6 = v_1^c.(w_5 + p) + v_2^c.(w_6 + q)$$
(4)

where v_1^c and v_2^c are the constants- their values don't change, as w_1, w_2, w_3, w_4 don't get updated.

During updation between the 0^{th} and 1^{st} iterations, dv_1 ' can have 3 possible values based on active and dead RELU before and after updation.

Consider a single input scheme, i.e., $[x_1,x_2]$, such that additional compensation to w_5 at each iteration is:

$$\frac{dE}{dw'_5} = p$$

Using the chain rule, we can generalize this to calculate the total compensation weight P that must be added at the n_{th} iteration. Here, p and P compensate for weights w_1, w_2 . Similarly, q and Q can be defined to compensate for weights w_3, w_4 . Let $w_5 + p$ be r_5 and $w_6 + q$ be r_6 . r may be interpreted as a transformation on W_2 that takes care of the contribution factor from W_1 . Hence, the general equation for updation after 'n' iterations is:

$$r_{5n} = w_{5n} + \frac{w_{5n}^3 (x_1^2 + x_2^2)}{3(v_{1c} + \sum_{1}^n \frac{dE}{dY_n} w_{5n} (x_1^2 + x_2^2)^2)} \quad (A_{n-1} > 0 \quad \&\& \quad A_n > 0)$$

$$= w_{5n} + \frac{w_{5n}^2}{2\eta \frac{dE}{dY_n} (v_{1c} + \sum_{1}^n \frac{dE}{dY_n} w_{5n} (x_1^2 + x_2^2))} \quad (A_{n-1} > 0 \quad \&\& \quad A_n < 0)$$

where $A_n = (w_{1c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{5n} x_1) x_1 + (w_{2c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{5n} x_2) x_2$

$$r_{6n} = w_{6n} + \frac{w_{6n}^3 (x_1^2 + x_2^2)}{3(v_{2c} + \sum_{1}^n \frac{dE}{dY_n} w_{6n} (x_1^2 + x_2^2)^2)} \quad (B_{n-1} > 0 \quad \&\& \quad B_n > 0)$$

$$= w_{6n} + \frac{w_{6n}^2}{2\eta \frac{dE}{dY_n} (v_{2c} + \sum_{1}^n \frac{dE}{dY_n} w_{6n} (x_1^2 + x_2^2))} \quad (B_{n-1} > 0 \quad \&\& \quad B_n < 0)$$

where $B_n = (w_{3c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{6n} x_1) x_1 + (w_{4c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{6n} x_2) x_2$

²Refer to Supplementary Material for the full derivation

Here subscript n represents value at the n^{th} iteration, and subscript c represents the constant value, i.e., the initial value which does not change across iterations.

We find that for multiple inputs, the entire input sequence must be stored to calculate the P and Q values. Therefore, in the cases of updating either (i) w_5 , w_6 by adding compensatory weights p, q, or (ii) updating the compensatory weights p, q for a fixed w_5 , w_6 , we use a single input to run a basic exploratory analysis experiment³.

To handle multiple inputs, we update s_1 , s_2 as follows:

Initially:
$$s_1 = w_1 x_1 + w_2 x_2$$
, $s_2 = w_3 x_1 + w_4 x_2$, $S_1 = w_1 x_1 - w_2 x_2$, $S_2 = w_3 x_1 - w_4 x_2$
Updated as: $s_1^{@'} = \frac{s_1' + S_1'}{2} x_1^{@} + \frac{s_1' - S_1'}{2} x_2^{@}$, $s_2^{@'} = \frac{s_2' + S_2'}{2} x_1^{@} + \frac{s_2' - S_2'}{2} x_2^{@}$
 $S_1^{@'} = \frac{s_1' + S_1'}{2} x_1^{@} - \frac{s_1' - S_1'}{2} x_2^{@}$, $S_2^{@'} = \frac{s_2' + S_2'}{2} x_1^{@} - \frac{s_2' - S_2'}{2} x_2^{@}$ (5)

Above, we have shown that in the context of 6, for weight layer W_2 , $(W_2 + p)$ can contribute (attend for text) equivalent to updated W_1 , and W_2 . p is the contribution/weight attention factor of W_1 to W_2 , i.e., the amount by which W_1 contributes/attends to W_2 . In this case, W_2 weights are now non-linear, and we find that W_2 weights can take the form $W_2 + W_2^3$. By introducing non-linear weights, we can therefore reduce the total number of weights by 1 set (i.e., the number of weights corresponding to a particular layer). By showing that NN of n layers can be collapsed to n-1 layers, we can further use the method of induction to theoretically prove that NN can be collapsed from (n-1)->(n-2)->...->1 layer, such that the final network has just non-linear weights for layer W_n .

This method of W_n calculation is the *Front Contribution Algorithm*, as instead of propagating error backwards, the network propagates contribution forward to collapse the network.

4 Conclusion:

We proposed a simple, novel algorithm, the Front-Contribution algorithm, as a compact alternative to BP. Our algorithm has several advantages:

(i) Front Contribution is a one-time calculation, i.e., given a NN structure, with number of layers, activations, etc., the algorithm will output the non-linear weights of a corresponding collapsed network. While the expression for W_n weights might be complex, the algorithm's time complexity remains O(1).

(ii) As the number of weights in a collapsed network is much lesser than in the original network, GPU space usage will be drastically improved. Deep Learning has significantly benefited from parallel computing, so by substantially improving parallel computing performance, we can potentially create deeper networks.

(iii) Due to the absence of back-propagation, we expect network training time to also be drastically reduced.

(iv) The post-training non-linear weights (represented in terms of their layers, W_i)– consider $W_n = W_n + k_1 W_n^3 + k_2 W_n^5 + ...$ – can potentially represent the features that the network has learned. Later terms in the preceding expression should represent simpler features (like edges), while initial terms should represent more complex features. However, W_n will give us an idea of the aggregate features that are important for NN. We define aggregate features as the final set of features that NN cares about, after attending to various features in a hierarchical manner.

(v) The modified network can always be expanded to a conventional network depending on our requirements– for example, we might want to see activations of attention neurons, and can therefore re-expand the network up till those neurons, without having to expand it fully.

(vi) This derivation can be modified and extended to other types of networks, such as CNNs, RNNs and transformers.

³Refer to Supplementary Materials: Experiments

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5 Supplemental Material

We know that a set of linear layers can be collapsed to a single layer network; however, non-linear activation functions have hitherto restricted the collapse of networks that they are applied in. Here, we justify that every fully connected multi-layer network can be collapsed to a single layer network, using the *Front Contribution Algorithm*, eliminating the requirement of backpropagation.

Analogy:

Consider an assembly line, where 10 people $(P_1 - P_{10})$ work sequentially to produce a toy car from plastic. P_1 cuts raw plastic (input) into different sizes, and gives it to P_2 , P_2 smooths the plastic and gives it to P_3 (intermediate output), and so on until P_{10} produces the toy car (final output). Initially, all the people do the task with no prior knowledge, so they perform random actions, aimed at producing the final output (car). P_{10} sees the output produced, compares it to the desired toy car,

and rectifies his actions based on the input he receives from P_9 , to make the output closer and closer to the toy car. P_9 , P_8 , etc. do the same- they look at the original error that they receive, find out how much they contribute to the error, and then try to rectify their actions by small amounts (not knowing exactly how much to rectify) to produce a perfect toy car. Therefore, every individual finds their respective individual actions to be performed and collectively produce a toy car. They can use this approach to build many other things, such as houses, fountains, etc., until they succeed as often as possible. Is this however an efficient method of learning to build anything?

Consider a person, P_{11} , who observes that P_2 is merely doing a fine-tuned version of P_1 's work by getting plastic to a desired physical state, and can therefore reliably do both P_1 's and their own work as they both are new to building. If P_1 is eliminated, and P_2 is asked to both cut and smooth plastic, by hiring one less person and reducing cost of transfer between P_1 and P_2 , the budget allows for extra training time for P_2 . P_2 also doesn't need to necessarily take more time to complete their work, if given a tool that both cuts and smooths the cut edges of plastic simultaneously– i.e., the output from P_2 becomes the sum of rectified effort from P_1 and P_2 . This is a one time cost, and P_{11} can develop the required tool by finding the 'contribution' of P_1 's actions to P_2 , i.e., how much P_1 's work attends to P_2 . Similarly, by observing the contributions of workers down the assembly line, P_{11} is able to eliminate all but the last worker, and provide him with a single tool that can be used to perform all the work. This results in a drastic decrease in budget and increase in building speed. It also allows more time for process and output analysis, to understand how different steps and features contribute towards a specific task and overall output– the process becomes more transparent, akin to process explainability.

This can be extended towards other concepts such as center of mass in physics as well. We aim to calculate a single-layer network with non-linear weights (calculated with Forward Contribution) whose performance is tantamount to a conventional multi-layer network that uses backpropagation. This is not an approximation (mimic) network, but an equivalent network to BP.

Formalization:

Let a neural network (NN) have n layers, such that each layer of its weights are represented as $W_i : i\epsilon[1, n]$, where W_1 is applied to the input and W_n is on the branch connected to the output node. Let the input of NN be X, intermediate layer outputs be $v_j : j\epsilon[1, n-1]$, and final output be Y. We can represent Y as:

$$Y = f(X, W_1, W_2, \dots, W_n)$$
(6)

Here, each of the weights are randomly initialized, and then updated using conventional back propagation. Using the chain rule, we have:

$$\Delta W_1 = f(W_2, W_3 \dots W_n) \tag{7}$$

$$\Delta W_2 = f(W_3, W_4 \dots W_n) \tag{8}$$

From equation 6, we see that the input X is multiplied with the weights, and then transformed by activations at various layers to produce the output Y. In other words, we can say that weight layers $W_1...W_n$ indirectly contribute towards deciding what Y will be for a given X, in a hierarchical sequence starting from W_1 till W_n . The definition of 'contribution' varies depending on the application–for example, in the case of language models like BERT, contribution refers to attention.

Now, from equations 7,8 we see that in the training process, for the i^{th} layer, Δ W is a function of all the W_i layers, from the $(i + 1)^{\text{th}}$ layer up till the n^{th} layer. From equation 7, we see that ΔW_1 is a function of $W_2...W_n$, but does not further depend on W_1 , i.e., the value of W_1 at any iteration of training depends on the initial value of W_1 - a random static value- and weights $W_2...W_n$. So, we can say that W_1 is not part of the system basis- the set of vectors that can be used to represent any system state, such as the use of x, y, z bases to represent any state of a 3D system- as $W_2...W_n$ can represent the output value at any iteration. So, vector W_1 is not actually a necessary variable.

Since W_1 is not part of the basis, it is actually not a necessary variable; for example, by defining a 3D system in terms of x, y, z, x + y + z, we utilize an unnecessary variable, x + y + z. As W_1 is not an independent weight layer, we are therefore wasting GPU space by using it. However, if the weight layer W_1 is ignored, will the system still function as desired? The value of v_1 depends on weights in W_1 , so if W_1 weights are not updated, v_2 will retain its old, incorrect value, consequently affecting Y even if all other weights are updated correctly, as the network is connected as a hierarchy.



Figure 2: Neural network structure for (a) Backpropagation, (b) Forward Contribution. Here, layer W_1 contains weights w_1, w_2, w_3, w_4 and W_2 contains weights w_5, w_6 .

How do we compensate the non-updation of weights in W_1 ? Let a compensation weight p be added to weights in W_2 such that Y remains the same, i.e., p compensates for the change in v_2 that normally happens with the updation of W_1 . Here, p must be a non-linear function of weights in W_2 , as it compensates for an update that depends on $W_2...W_n$. p can be derived by equating the v_2 value found post updation of W_1 in conventional backpropgation, and the v_2 value calculated without updating W_1 and replacing weights in W_2 as $W_2 + p$.

6 Finding an Expression for *p*

Let X be the input vector: $[[x_1,x_2], [x_1^{@},x_2^{@}], [x_1^*,x_2^*], [x_1^{\#}, x_2^{\#}]]$. Let Y_g be the vector of gold labels: $[y_g, y_g^{@}, y_g^*, y_g^{\#}]$. Let w_1, w_2, w_3, w_4 be the weights of the first layer (W_1) . Let w_5, w_6 be the weights of the last layer (W_2) . We define the state of the intermediate nodes v_1 and v_2 for a given input as:

$$s_1 = w_1 \cdot x_1 + w_2 \cdot x_2, s_2 = w_3 \cdot x_1 + w_4 \cdot x_2 \tag{9}$$

The activation function for intermediate nodes is RELU, i.e.:

$$v_1 = RELU(s_1), v_2 = RELU(s_2) \tag{10}$$

where

$$RELU(s) = \begin{cases} s & \text{if } s > 0\\ 0 & \text{if } s < 0 \end{cases}$$

Let η be the learning rate and Y be the output vector: $[y, y^{@}, y^{*}, y^{\#}]$, where

$$y = w_5 \cdot v_1 + w_6 \cdot v_2 \tag{11}$$

Let the error E be defined as:

$$E = \frac{1}{2}(y - y_g)^2, \text{ i.e., } \frac{dE}{dY} = y - y_g$$
(12)

The following repeats successively for each epoch, for a single given input:

$$w_{5}' = w_{5} - \eta \frac{dE}{dY} v_{1}, w_{6}' = w_{6} - \eta \frac{dE}{dY} v_{2}$$
(13)

 $\frac{dE}{dw_1}$ can have 2 possible values based on whether the RELU is dead or not, as can $\frac{dE}{dw_2}, \frac{dE}{dw_3}$ and $\frac{dE}{dw_4}$:

$$\frac{dE}{dw_1} = 0 \text{ (or) } \frac{dE}{dw_1} = \frac{dE}{dY} \cdot w_5 \cdot x_1(s_1 > 0)$$
$$\frac{dE}{dw_2} = 0 \text{ (or) } \frac{dE}{dw_2} = \frac{dE}{dY} \cdot w_5 \cdot x_2(s_1 > 0)$$
$$\frac{dE}{dw_3} = 0 \text{ (or) } \frac{dE}{dw_3} = \frac{dE}{dY} \cdot w_6 \cdot x_1(s_2 > 0)$$
$$\frac{dE}{dw_4} = 0 \text{ (or) } \frac{dE}{dw_4} = \frac{dE}{dY} \cdot w_6 \cdot x_2(s_2 > 0)$$

Based on this, we update the weights of the first layer as:

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$$w_{1}' = w_{1} - \eta \frac{dE}{dY} w_{5}' x_{1}(s_{1} > 0)(\text{or}) w_{1}' = w_{1}(s_{1} < 0)$$

$$w_{2}' = w_{2} - \eta \frac{dE}{dY} w_{5}' x_{2}(s_{1} > 0)(\text{or}) w_{2}' = w_{2}(s_{1} < 0)$$

$$w_{3}' = w_{3} - \eta \frac{dE}{dY} w_{6}' x_{1}(s_{2} > 0)(\text{or}) w_{3}' = w_{3}(s_{2} < 0)$$

$$w_{4}' = w_{4} - \eta \frac{dE}{dY} w_{6}' x_{2}(s_{2} > 0)(\text{or}) w_{4}' = w_{4}(s_{2} < 0)$$

$$(14)$$

$$s'_{1} = w'_{1} \cdot x_{1} + w'_{2} \cdot x_{2}, s'_{2} = w'_{3} \cdot x_{1} + w'_{4} \cdot x_{2},$$

$$v'_{1} = RELU(s'_{1}), v'_{2} = RELU(s'_{2}), y = w'_{5} \cdot v'_{1} + w'_{6} \cdot v'_{2}$$
(15)

where

$$\frac{dE}{dY} = y - y_g = (w_5 \cdot v_1 + w_6 \cdot v_2) - y_g \tag{16}$$

From equations 14, 15:

$$s_1' = (w_1 - \eta \frac{dE}{dY} w_5' x_1) x_1 + (w_2 - \eta \frac{dE}{dY} w_5' x_2) x_2$$
(17)

$$= w_1 \cdot x_1 + w_2 \cdot x_2 - \eta \frac{dE}{dY} w_5'(x_1^2 + x_2^2)$$
$$= s_1 - \eta \frac{dE}{dY} w_5'(x_1^2 + x_2^2)$$

Parallely,
$$s'_{2} = s_{2} - \eta \frac{dE}{dY} w'_{6} (x_{1}^{2} + x_{2}^{2})$$

If the next input is $[x_1^{@}, x_2^{@}]$ then for the next epoch, we have:

$$s_{1}^{\prime @} = (w_{1} - \eta \frac{dE}{dY} w_{5}^{\prime} x_{1}) x_{1}^{@} + (w_{2} - \eta \frac{dE}{dY} w_{5}^{\prime} x_{2}) x_{2}^{@}$$
(18)
$$= w_{1} \cdot x_{1}^{@} + w_{2} \cdot x_{2}^{@} - \eta \frac{dE}{dY} w_{5}^{\prime} (x_{1} x_{1}^{@} + x_{2} x_{2}^{@})$$

$$s_{2}^{\prime @} = (w_{3} - \eta \frac{dE}{dY} w_{6}^{\prime} x_{1}) x_{1}^{@} + (w_{4} - \eta \frac{dE}{dY} w_{6}^{\prime} x_{2}) x_{2}^{@}$$
$$= w_{3} \cdot x_{1}^{@} + w_{4} \cdot x_{2}^{@} - \eta \frac{dE}{dY} w_{6}^{\prime} (x_{1} x_{1}^{@} + x_{2} x_{2}^{@})$$

Using $s_1^{@}=w_1 \cdot x_1^{@}+w_2 \cdot x_2^{@}$, $s_2^{@}=w_3 \cdot x_1^{@}+w_4 \cdot x_2^{@}$, and equation 17, we can simplify equation 18 as:

$$s_{1}^{\prime @} = s_{1}^{@} + \frac{s_{1}^{\prime} - s_{1}}{x_{1}^{2} + x_{2}^{2}} (x_{1}x_{1}^{@} + x_{2}x_{2}^{@})$$

$$s_{2}^{\prime @} = s_{2}^{@} + \frac{s_{2}^{\prime} - s_{2}}{x_{1}^{2} + x_{2}^{2}} (x_{1}x_{1}^{@} + x_{2}x_{2}^{@})$$
(19)

Let:

$$s_{1}^{*} = w_{1} \cdot x_{1}^{*} + w_{2} \cdot x_{2}^{*}, s_{1}^{\#} = w_{1} \cdot x_{1}^{\#} + w_{2} \cdot x_{2}^{\#}$$
$$s_{2}^{*} = w_{3} \cdot x_{1}^{*} + w_{4} \cdot x_{2}^{*}, s_{2}^{\#} = w_{3} \cdot x_{1}^{\#} + w_{4} \cdot x_{2}^{\#}$$

Using the above with equation 19, for the next 2 epochs with inputs $[x_1^*, x_2^*]$ and $[x_1^\#, x_2^\#]$, we get:

If w_1, w_2, w_3, w_4 are not updated, then non-linear 'compensation weights' of p, q must be added to w_5, w_6 respectively in order to preserve Y output updation. To derive values of p, q, consider that for the initial updation, using equations 9, 10, 11, 13 we can say that:

$$Y_{backpropgation} = Y_{contribution factor} \implies v_1.w_5 + v_2.w_6 = v_1^c.(w_5 + p) + v_2^c.(w_6 + q)$$
(21)

where v_1^c and v_2^c are the constants– their values don't change, as w_1, w_2, w_3, w_4 don't get updated. So, for the first updation, we get:

$$\begin{aligned} v_1'.w_5' + v_2'.w_6' &= v_1^c.(w_5' + p) + v_2^c.(w_6' + q) \\ \Longrightarrow & (v_1^c + dv_1').w_5' + (v_2^c + dv_2').w_6' = v_1^c.(w_5' + p) + v_2^c.(w_6' + q) \end{aligned}$$

where w'_5, w'_6 are updated as per equation 13. Now, compensating v_1 and v_2 separately, we have:

$$(v_1^c + dv_1').w_5' = v_1^c.(w_5' + p) \implies dv_1'.w_5' \qquad = v_1^c.p \tag{22}$$

Similarly,

$$(v_2^c + dv_2').w_6' = v_2^c.(w_6' + q) \implies dv_2'.w_6' = v_2^c.q$$

During updation between the 0th and 1st iterations, dv_1 ' can have 3 possible values: (i) RELU is dead at the 0th iteration and stays dead after the 1st update, (ii) RELU is active at the 0th iteration and stays active after the 1st update, and (iii) RELU is dead at the 0th iteration and becomes active after the 1st update:

$$dv_1' = 0 \tag{23}$$

$$\begin{aligned} &dv_1' = dw_1'.x_1 + dw_2'.x_2 \ \left((s_1 \ge 0) \&\&(s_1' \ge 0)\right) \\ &dv_1' = -v_1 = -s_1 \ \left((s_1 \ge 0) \&\&(s_1' < 0)\right) \end{aligned}$$

where

$$s_1 = w_1 \cdot x_1 + w_2 \cdot x_2$$

$$s'_1 = s_1 (s_1 < 0)$$

i.e.,

$$s_1' = (w_1' \cdot x_1 + w_2' \cdot x_2)(s_1 \ge 0)$$
(24)

$$\implies s_1' = (w_1 - \eta \frac{dE}{dw_1}) \cdot x_1 + (w_2 - \eta \frac{dE}{dw_2}) \cdot x_2$$

$$\implies s_1' = (w_1 - \eta \frac{dE}{dY} \cdot w_5' \cdot x_1 (s_1 \ge 0)) \cdot x_1 + (w_2 - \eta \frac{dE}{dY} \cdot w_5' \cdot x_2 (s_1 \ge 0)) \cdot x_2$$

$$\implies s_1' = s_1 - \eta \frac{dE}{dY} \cdot w_5' (x_1^2 + x_2^2) (s_1 \ge 0)$$

Using equation 24 in 23,

$$dv'_{1} = -\eta \frac{dE}{dY} \cdot w'_{5}(x_{1}^{2} + x_{2}^{2})(((w_{1}.x_{1} + w_{2}.x_{2}) \ge 0)\&\&$$

$$((w_{1} - \eta \frac{dE}{dY} \cdot w'_{5}.x_{1}).x_{1} + (w_{2} - \eta \frac{dE}{dY} \cdot w'_{5}.x_{2}).x_{2})) \ge 0))$$

$$dv'_{1} = -(w_{1}.x_{1} + w_{2}.x_{2})(((w_{1}.x_{1} + w_{2}.x_{2}) \ge 0)\&\&$$

$$((w_{1} - \eta \frac{dE}{dY} \cdot w'_{5}.x_{1}).x_{1} + (w_{2} - \eta \frac{dE}{dY} \cdot w'_{5}.x_{2}).x_{2})) < 0))$$
(25)

Using equation 22 and 26, we get:

$$p = \frac{dv'_1 \cdot w'_5}{v_1^c}$$

$$= \frac{-\eta \frac{dE}{dY} \cdot w'_5 (x_1^2 + x_2^2) (((w_1 \cdot x_1 + w_2 \cdot x_2) \ge 0) \& \& ((w_1 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_1) \cdot x_1 + (w_2 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_2) \cdot x_2)) \ge 0)) \cdot w'_5}{v_1^c}$$

$$= \frac{-\eta \frac{dE}{dY} \cdot w'_5 (x_1^2 + x_2^2) (((w_1 \cdot x_1 + w_2 \cdot x_2) \ge 0) \& \& ((w_1 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_1) \cdot x_1 + (w_2 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_2) \cdot x_2)) \ge 0))}{v_1^c}$$

$$= \frac{-\eta \frac{dE}{dY} \cdot w'_5 (x_1^2 + x_2^2) (((w_1 \cdot x_1 + w_2 \cdot x_2) \ge 0) \& \& ((w_1 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_1) \cdot x_1 + (w_2 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_2) \cdot x_2)) \ge 0))}{v_1^c}$$

$$= \frac{-\eta \frac{dE}{dY} \cdot w'_5 (x_1^2 + x_2^2) (((w_1 \cdot x_1 + w_2 \cdot x_2) \ge 0) \& \& ((w_1 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_1) \cdot x_1 + (w_2 - \eta \frac{dE}{dY} \cdot w'_5 \cdot x_2) \cdot x_2)) \ge 0))}{v_1^c}$$

$$p = \frac{dv'_{1}.w'_{5}}{v_{1}^{2}}$$

$$= \frac{-(w_{1}.x_{1}+w_{2}.x_{2})(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta\frac{dE}{dY}.w'_{5}.x_{1}).x_{1}+(w_{2}-\eta\frac{dE}{dY}.w'_{5}.x_{2}).x_{2}))\geq 0)).w'_{5}}{v_{1}^{2}}$$

$$= \frac{-v_{1}^{c}.(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta\frac{dE}{dY}.w'_{5}.x_{1}).x_{1}+(w_{2}-\eta\frac{dE}{dY}.w'_{5}.x_{2}).x_{2}))\geq 0)).w'_{5}}{v_{1}^{2}}$$

$$= -w'_{5}.(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta\frac{dE}{dY}.w'_{5}.x_{1}).x_{1}+(w_{2}-\eta\frac{dE}{dY}.w'_{5}.x_{2}).x_{2}))\geq 0))$$

$$(27)$$

The above represents the amount of weight p to be added for the given iteration. Consider only a single input scheme, i.e., $[x_1, x_2]$, such that additional compensation to w'_5 at each iteration is:

$$\frac{dE}{dw_5'} = p$$

Then using the chain rule, we can generalize this to calculate the total compensation weight P that must be added at the n_{th} iteration as follows:

$$Additional - \eta \frac{dE}{dY} \cdot v_1^c \cdot \frac{dP}{dw_5} = p$$

$$\implies \frac{dP}{dw_5} = \frac{-p}{\eta \frac{dE}{dY} \cdot v_1^c}$$

$$\implies P = \int \frac{-p}{\eta \frac{dE}{dY} \cdot v_1^c} \cdot dw_5$$
(28)

We can find P in terms of w_5 ' using equation 18, 20 as:

$$=\int \frac{\frac{\eta \frac{dE}{dY} \cdot w_{5}'(x_{1}^{2}+x_{2}^{2})(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta \frac{dE}{dY}.w_{5}'.x_{1}).x_{1}+(w_{2}-\eta \frac{dE}{dY}.w_{5}'.x_{2}).x_{2}))\geq 0))}{\eta \frac{dE}{dY} \cdot w_{1}' \cdot w_{1}'} dw_{5}'$$

$$=\int \frac{w_{5}'^{2}(x_{1}^{2}+x_{2}^{2})(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta \frac{dE}{dY}.w_{5}'.x_{1}).x_{1}+(w_{2}-\eta \frac{dE}{dY}.w_{5}'.x_{2}).x_{2}))\geq 0))}{v_{1}^{c^{2}}} dw_{5}'$$

$$=\frac{w_{5}'^{3}(x_{1}^{2}+x_{2}^{2})(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta \frac{dE}{dY}.w_{5}'.x_{1}).x_{1}+(w_{2}-\eta \frac{dE}{dY}.w_{5}'.x_{2}).x_{2}))\geq 0))}{3v_{1}^{c^{2}}} dw_{5}'$$

$$(29)$$

and using equation 19, 20 as:

$$= \int \frac{w_{5}'(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta\frac{dE}{dY}.w_{5}'.x_{1}).x_{1}+(w_{2}-\eta\frac{dE}{dY}.w_{5}'.x_{2}).x_{2}))\geq 0))}{\eta\frac{dE}{dY}.v_{1}^{c}} .dw_{5}'$$

$$= \frac{w_{5}'^{2}(((w_{1}.x_{1}+w_{2}.x_{2})\geq 0)\&\&((w_{1}-\eta\frac{dE}{dY}.w_{5}'.x_{1}).x_{1}+(w_{2}-\eta\frac{dE}{dY}.w_{5}'.x_{2}).x_{2}))\geq 0))}{2\eta\frac{dE}{dY}.v_{1}^{c}}$$
(30)

Here, p and P compensate for weights w_1, w_2 . Similarly, q and Q can be defined to compensate for weights w_3, w_4 . Let $w_5 + p$ be r_5 and $w_6 + q$ be r_6 . r may be interpreted as a transformation on w_{layer2} that takes care of the contribution factor from w_{layer1} . Hence, the general equation for updation after 'n' iterations is:

$$r_{5n} = w_{5n} + \frac{w_{5n}^3 (x_1^2 + x_2^2)}{3(v_{1c} + \sum_{1}^n \frac{dE}{dY_n} w_{5n} (x_1^2 + x_2^2)^2)} \quad (A_{n-1} > 0 \quad \&\& \quad A_n > 0)$$

$$= w_{5n} + \frac{w_{5n}^2}{2\eta \frac{dE}{dY_n} (v_{1c} + \sum_{1}^n \frac{dE}{dY_n} w_{5n} (x_1^2 + x_2^2))} \quad (A_{n-1} > 0 \quad \&\& \quad A_n < 0)$$

where $A_n = (w_{1c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{5n} x_1) x_1 + (w_{2c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{5n} x_2) x_2$

$$r_{6n} = w_{6n} + \frac{w_{6n}^2 (x_1^2 + x_2^2)}{3(v_{2c} + \sum_{1}^n \frac{dE}{dY_n} w_{6n} (x_1^2 + x_2^2)^2)} \quad (B_{n-1} > 0 \quad \&\& \quad B_n > 0)$$

$$= w_{6n} + \frac{w_{6n}^2}{2\eta \frac{dE}{dY_n} (v_{2c} + \sum_{1}^n \frac{dE}{dY_n} w_{6n} (x_1^2 + x_2^2))} \quad (B_{n-1} > 0 \quad \&\& \quad B_n < 0)$$

where $B_n = (w_{3c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{6n} x_1) x_1 + (w_{4c} + \sum_{1}^{n} \frac{dE}{dY_n} w_{6n} x_2) x_2$

Here subscript n represents value at the n^{th} iteration, and subscript c represents the constant value, i.e., the initial value which does not change across iterations.

In the condition $s_1 \ge 0$ and $s'_1 \ge 0$, using equations 13, 21, 18, for initial input $[x_1, x_2]$ we have:

$$p = -\eta \frac{dE}{dy} w_5'^2 \frac{(x_1^2 + x_2^2)}{v_1^c}, \ q = -\eta \frac{dE}{dy} w_6'^2 \frac{(x_1^2 + x_2^2)}{v_2^c}$$
$$y' = (w_5' + p)v_1^c + (w_6' + p)v_2^c, \ \frac{dE'}{dy} = y' - y_g$$
$$w_5'' = w_5' - \eta \frac{dE'}{dy} v_1' = w_5' - \eta (v_1^c w_5' + v_1^c p + v_2^c w_6' + v_2^c q - y_g)(v_1^c - pv_1^c)$$
$$w_6'' = w_6' - \eta \frac{dE'}{dy} v_1' = w_5' - \eta (v_1^c w_5' + v_1^c p + v_2^c w_6' + v_2^c q - y_g)(v_2^c - qv_2^c)$$

For next input $[x_1^{@}, x_2^{@}]$ we have:

$$v_{1}^{@'}w_{5}' = (w_{5}' + p + p^{@})v_{1}^{@}$$

$$\implies -\eta \frac{dE}{dy}w_{5}'^{2}(x_{1}x_{1}^{@} + x_{2}x_{2}^{@}) = p^{@}(w_{1}x_{1}^{@} + w_{2}x_{2}^{@}) + p(w_{1}x_{1}^{@} + w_{2}x_{2}^{@})$$

$$p^{@} = p - \eta \frac{dE}{dy}w_{5}'^{2}\frac{(x_{1}x_{1}^{@} + x_{2}x_{2}^{@})}{v_{1}'^{@}}$$
Parallely, $q^{@} = q - \eta \frac{dE}{dy}w_{6}'^{2}\frac{(x_{1}x_{1}^{@} + x_{2}x_{2}^{@})}{v_{2}'^{@}}$
(31)

From the above, we see that for multiple inputs, the entire input sequence must be stored to calculate the P and Q values. Therefore, in the cases of updating either (i) w_5 , w_6 by adding compensatory weights p, q, or (ii) updating the compensatory weights p, q for a fixed w_5 , w_6 , we only use one input.

To handle multiple inputs, we update s_1 , s_2 as follows:

Initially:

$$s_1 = w_1 x_1 + w_2 x_2, \ s_2 = w_3 x_1 + w_4 x_2$$

$$S_1 = w_1 x_1 - w_2 x_2, \ S_2 = w_3 x_1 - w_4 x_2$$

Updated as:

$$s_{1}^{\ @'} = \frac{s_{1}' + S_{1}'}{2} x_{1}^{\ @} + \frac{s_{1}' - S_{1}'}{2} x_{2}^{\ @}, \ s_{2}^{\ @'} = \frac{s_{2}' + S_{2}'}{2} x_{1}^{\ @} + \frac{s_{2}' - S_{2}'}{2} x_{2}^{\ @}$$
$$S_{1}^{\ @'} = \frac{s_{1}' + S_{1}'}{2} x_{1}^{\ @} - \frac{s_{1}' - S_{1}'}{2} x_{2}^{\ @}, \ S_{2}^{\ @'} = \frac{s_{2}' + S_{2}'}{2} x_{1}^{\ @} - \frac{s_{2}' - S_{2}'}{2} x_{2}^{\ @}$$
(32)

Above, we have shown that in the context of 6, (W2 + p) (where W_2 is the second weight layer comprising of w_5 , w_6) can contribute (attend for text) equivalent to updated W_1 , and W_2 . p is the contribution/weight attention factor of W_1 to W_2 , i.e., the amount by which W_1 contributes/attends to W_2 . In this case, W_2 weights are now non-linear–based on equation 30, W_2 weights can take the form of $W_2 + W_2^3$. By introducing non-linear weights, we can therefore reduce the total number of weights by 1 set (i.e., the number of weights corresponding to a particular layer). By showing that NN of n layers can be collapsed to n-1 layers, we can further use the method of induction to theoretically prove that NN can be collapsed from (n-1) - > (n-2) - > ... - > 1 layer, such that the final network has just non-linear weights for layer W_n .

This method of W_n calculation is the *Front Contribution Algorithm*, as instead of propagating error backwards, the network propagates contribution forward to collapse the network.

Experiments:

The results of the three techniques we use for Forward Contribution, over the implementation of the XOR task are displayed in Figure 3.

The order of error is $< 10^{-15}$, proving that Forward Contribution is a true equivalent of backpropagation.



Figure 3: Here in all 3 figures, x-axis represents the training iteration, and the y-axis represents the error between output produced through backpropagation and the ouptut produced by forward contribution. Top: On updation of weights w_5, w_6 as $w_5 + p, w_6 + q$. Middle: On updation of values of p, q without updation of w_5, w_6 . Bottom: Updation of states v_1, v_2 without weight updation.

Backpropagation vs. Front Contribution Illustrations: We illustrate the process flows of Back Propagation and Front Contribution in Figure 4.



Figure 4: On the left hand side, from top->bottom are the steps for Back-Propagation; the top right image has the final summary of Back-Propagation. The remaining images on the right show how Front-Contribution functions (top->bottom).